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How to teach geometry theorems using worked examples: A cognitive load theory perspective

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Abstract. Some geometry theorems are studied at Year 7, such as angles formed by parallel lines and a transversal line. There are at least nine theorems should be simultaneously considered to solve a complex problem-solving in this topic. For example, a problem about angles formed by parallel lines and more than one transversal lines. Many students often have difficulties when solving the problem in this topic because it requires a good understanding of angle representation. A Cognitive Load Theory (CLT) has suggested that learning complex problem solving is more efficient for novices when worked examples are presented. Worked Examples (WE) are suggested for novice learners because it can reduce the extraneous cognitive load. Hence, working memory with limited capacity and duration can construct knowledge effectively. However, there are issues when designing WE for most geometry problems, such as the split attention. This paper presents some WE of angles formed by parallel lines and a transversal lines in several theorems. This paper discusses further how to teach the WE and propose four learning steps: (1) apperception, (2) students solve geometry problems with worked example strategies, (3) students present the results of work, and (4) teachers guide students to conclude learning outcomes.

1. Introduction

Problem-solving is an issue that is always interesting to be investigated. "Problem-solving should be the focus of school mathematics in the 1980s", is a message delivered by NCTM [1]. The message had a major impact in Japan, so from then on began to apply problem-solving in mathematics learning. The Netherlands also applied problem-solving in mathematics learning based on the principles of realistic mathematics education [2]. The main challenge in implementing problem-solving in the Netherlands is how to design good, unusual and new problem-solving problems for students. While in Singapore, problem-solving began to be applied in the curriculum of mathematics education since 1990 [3]. The phenomena occurring in these countries illustrate the importance of problem-solving skills in the mathematics learning that students must possess. When solving a problem, there are several ways, thinking, reasoning, working procedures, and even sense that can be used in planning the action. Often the methods used successfully in the process and so-called problem-solving strategies. Polya [4] mentions that there are many reasonable ways to solve problems. The skill at choosing a strategy is best learned by solving many problems. A partial list of strategies is included (1) look for a pattern, (2) examine related problems and determine if the same technique can be applied, (3) examine a simpler or special case of the problem to gain insight into the solution of the original problem, (4) make a table, (5) make a diagram, (6) write an equation, (7) use a guess and check, (8) work backwards, and (9) identify a subgoal. Not all of these strategies are used at once when solving a problem. It depends on the needs of either the problem itself or the students. One problem with another problem will require a different solving strategy. Similarly, one student with another student also has a different point of view when solving the problem. How students solve problems in mathematics learning can be applied in everyday life. However, the problem presented in the lesson is not always a word problem, but also can be a geometry problem, such as in Figure 1.

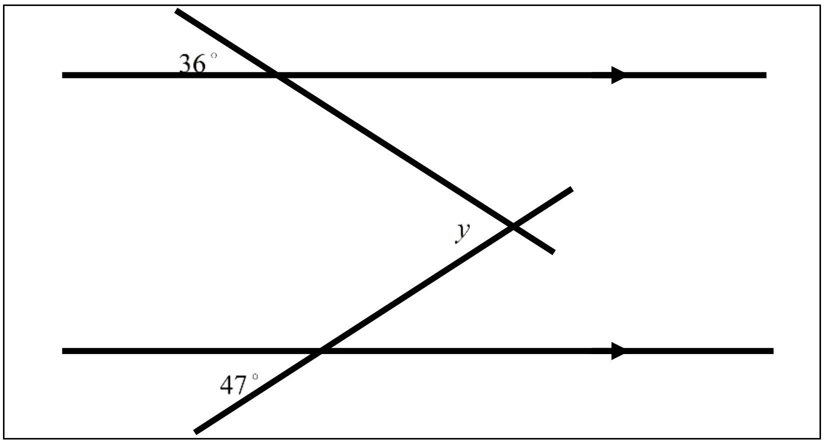

Figure 1. Example of a complex problem in geometry topic.

Tabel 1. Some angle theorems.

No	Theorems
1	The full rotation angles
2	The supplement angles
3	The complement angles
4	The opposite angles
5	The corresponding angles
6	The consecutive interior/exterior angles
7	The alternate interior/exterior angles
8	The angles in a triangle
9	The angles in a quadrilateral

Usually, in grade 7 or 8, students learn theorems related to angles formed by two parallel lines and transversal lines, such as in table 1. The material is a new material that students learn, so we can say novice learners. Students learn the theorems to solve easy problems to complex problems. Figure 1 is an example of a complex problem that is difficult for students. This is because the problem does not present explicitly the angles formed by two parallel lines and transverse lines, so in the solution, it takes certain strategies that are considered difficult for the novice learner. Most students have difficulties when solving the problem of angles formed by two parallel lines and transverse lines according to Van Hiele geometry theory because students are stuck at the visualization stage or level 0 [5]. While the given problem is at a higher level. If they are given a complex problem, the ways commonly used is the conventional way or trial and error. Previous research indicated that the knowledge obtained by students through conventional problem solving is not effective because it is influenced by the mean ends analysis [6], a general problem-solving strategy used when the prior knowledge is still lacking [7]. Cognitive load theorists suggest that learners without sufficient knowledge base, they will have difficulty to discover the new knowledge [8]. Therefore, novice

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learners require assistance or scaffolding to build a knowledge base relevant to be applied in the problem-solving task to create a productive learning environment for them [9].

Cognitive Load Theory (CLT) is an instructional theory based on our knowledge of human cognitive architecture [10]. Understanding students' cognitive processes while studying mathematics is one of the most important approaches to teaching mathematics effectively [11]. Working memory has the limited capacity [12] and duration [13] when used to process new information [7]. Limitations that will cause cognitive load, such as intrinsic, extraneous, and germane cognitive load. The intrinsic cognitive load is due to the degree of complexity of the information or material being studied. The extraneous cognitive load is caused by procedures or techniques of material presentation that are not in accordance with the learning activities so as not to contribute to the formation of the scheme. The germane cognitive load is caused by the number of mental efforts a person pours into the learning process or activities that can assist in the process of forming the scheme [14]. Therefore, by organizing the cause of intrinsic cognitive load, reducing the sources of extraneous cognitive load, and managing germane cognitive load, it is hoped that the learning activities will be better. CLT suggests the presentation of a Worked Example (WE) to teach complex problem solving for a novice learner. Worked examples are an example of the teacher's knowledge in long-term memory that can be seen and learned by students to acquire new knowledge [15]. Therefore, the problem-solving process will be more focused and reduce students cognitive load. So it will be more effective than unfocused problem-solving. So, the WE of complex problems in geometry and how to teach the students using the WE will be discussed further.

2. Worked examples for a geometry problem

Researchers have demonstrated that when learning the novel material, guided instruction through worked examples is more effective for novice learners than conventional problem-solving strategies [15-19]. As explained above that students will learn new geometry problems, so the worked example strategy is appropriate for learning. By providing novices with worked examples to study, their attention can be fully devoted to learning how the problem should be solved [7] rather than focus on solving the problem [20]. Therefore, the worked example can reduce extraneous cognitive load [15], then learning is enhanced through the construction of new schematic knowledge [6, 21]. Several studies using worked example to solve geometry problems [18, 19, 22] have been done by researchers. In addition, the design worked example to study geometry problems [9] also began to be developed.

None defines precisely the working example but there are several different types [16] at least consists of the problem statement, the goal of the problem, and the steps of solutions [6, 16]. The problem statement of this worked example is how a student can solve a complex geometry problem about angles formed by parallel lines and more than one transversal lines. Then the goal of the problem is students must calculate the angle formed by the intersection of two transverse lines that intersect the parallel lines as in Figure 1. The steps of solution to solve geometry problem in this worked example begin with the use of an auxiliary line. Then, students use the theorems that have been studied at each step. The number of steps depends on the number of theorems used. In addition, when creating worked example for geometry, there are several things that must be considered include how to show the visualization, how to format the picture, and how to explain without escalating the extraneous cognitive load [9].

Five pairs of worked examples and problems are presented in the worksheet to help students solve geometry problems as in figure 1. The five worked models presented have characteristics that can be classified into three types: worked example with four theorems, three theorems, and two theorems. Nevertheless, there is one characteristic in all worked examples that is the use of a dotted line to manipulate the diagram so that it can be solved using the theorems corresponding to the angles formed by two parallel lines and the transverse lines.

a) Worked example with four theorems

An example of solving a geometry problem using four theorems is the first and the second example presented in the worksheet. Although many of the steps used to solve both problems are the same, the diagrams are presented differently so that the theorems used are also different. The presented of worked example can be seen in Figure 2. The theorem about the opposite angles, the alternate interior angles, the angles in a triangle, and the supplement angles are used to solve the problem as in Figure 2 (a). As for Figure 2 (b) use the theorem about the supplement angles, the alternate interior angles, the opposite angle, and the angles in a triangle. In addition, in these worked examples, students are shown how to use auxiliary line (line extension in red) to make clear the angle formation or the transverse line.

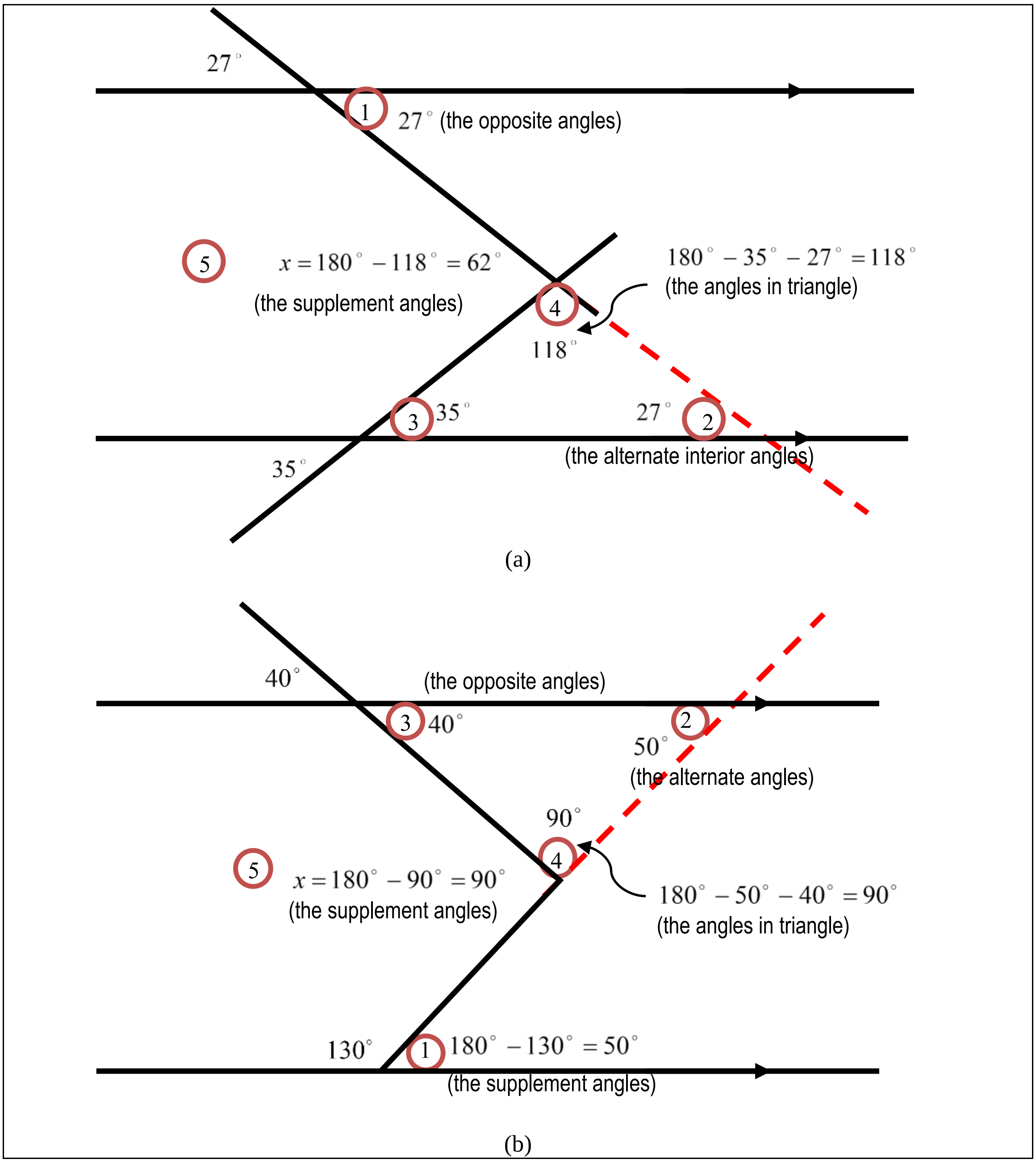

Figure 2. Two worked examples of using four angle theorems.

b) Worked example with three theorems

The three theorems used in the worked example illustration can be seen in Figure 3 (a) i.e. the theorem about the supplement angles, the complement angles, and the angles in the quadrilateral. In the example also requires the concept of a perpendicular line to form the angle that is 90º. This line (in red) is used to illustrate the transverse line, which is made as perpendicular to the parallel lines. While in Figure 3 (b) showing how to create auxiliary lines parallel to the parallel lines, which can also be an alternative. In this example, the theorem used is the alternate interior angles, consecutive interior angles, and the full rotation angles.

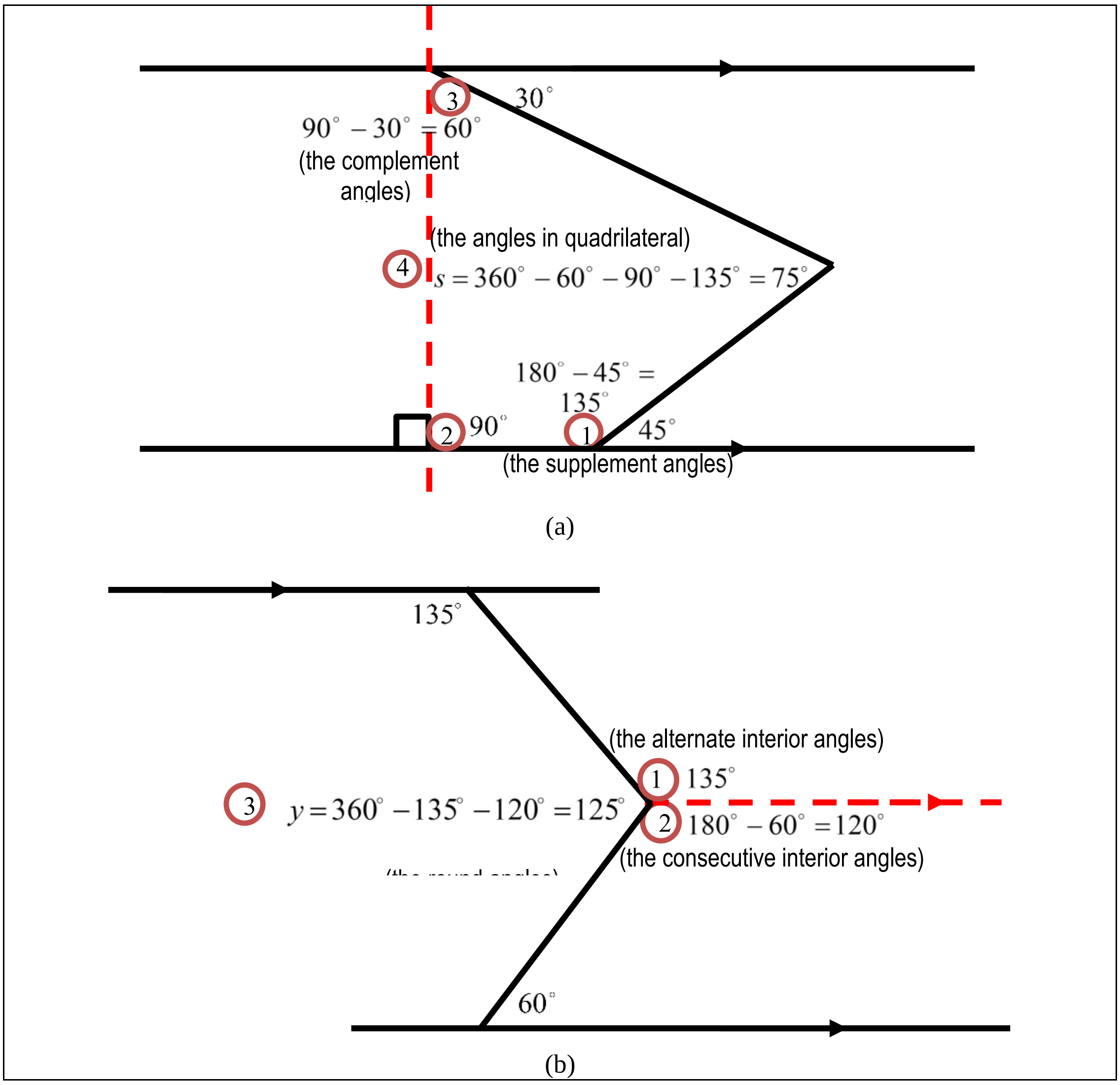

Figure 3. Two worked examples of using three angle theorems.

c) Worked example with two theorems

The last worked example used to solve a geometry problem uses only two theorems, consecutive interior angles, and full rotation angles. Auxiliary lines are used to manipulate diagrams, parallel to parallel lines. The working example presentation seen in Figure 4 could be the easiest and most convenient example when used to solve geometry problems. Nevertheless, often students should put some effort to figure it out conciously. They should generate understanding that such line can be a very important strategy for making manipulation when facing similar geometry problems.

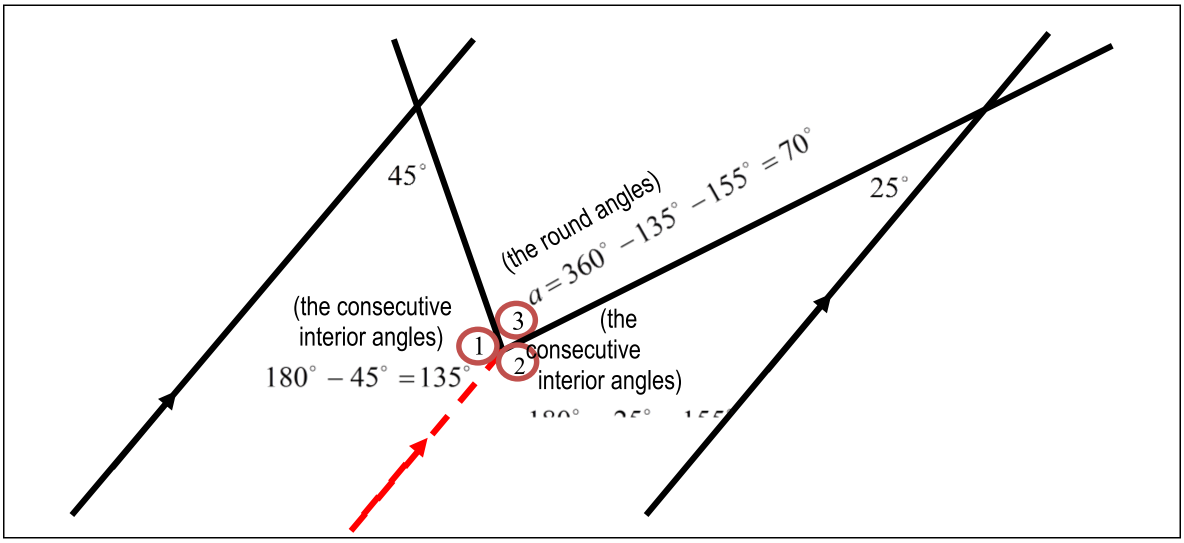

Figure 4. Worked example with two theorems

3. The Sequence of worked example based instruction

Worked example presented in the worksheet that is used as a medium for studying the problems of the angle formed by parallel lines and transverse lines. There are four learning steps that can be used to teach students to solve the problem with a worked example strategy, that is namely (1) apperception, (2) students solve geometry problems with worked example strategies, (3) students present the results of work, and (4) teachers guide students to conclude learning outcomes.

3.1. Apperception

Apperception given to the students is presented in the worksheet. Apperception is presented using a worked example strategy with the technique of pairing examples and problems on a single page. In order for students to use the worksheet well, the instructions should be clear. Students should study apperception independently. In addition, after students understand the example given, students should work on the problem **without** looking at an example.

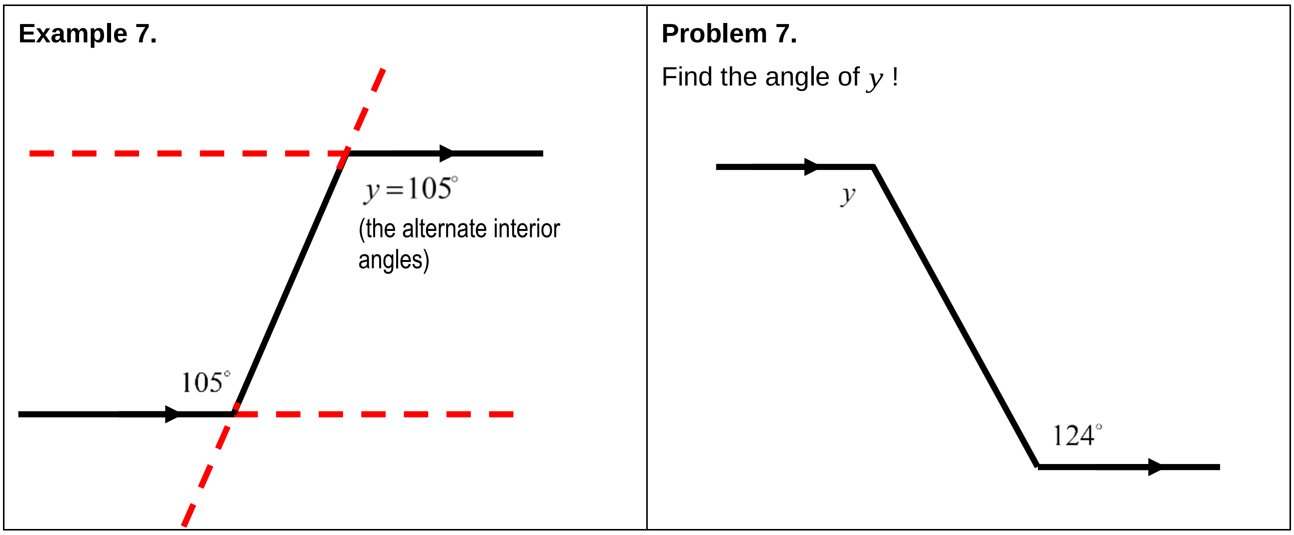

Figure 5. Example of apperception material in the worksheet.

There are nine angle theorems that must be understood by the students before solving a relevant geometry problem: (1) the full rotation angles, (2) the supplement angles, (3) the complement angles, (4) the opposite angles, (5) the corresponding angles, (6) the consecutive interior angles, (7) the alternate interior angles, (8) the angles in triangle, and (9) the angles in quadrilateral. Students must have already studied the theorems. Therefore, in previous learning, students have been taught the theorems and have proven it. Apperception activities mainly aim to recall students' memories of the theorems. The presentation of apperception does not merely remind students of the theorems. As in Figure 1, the students began to be introduced to the forms that have been modified to assist them in recognizing the relationship between the angle in a more complex problem. Integrating new information with old information that is already understood may assist in making sense of the new information [21]. Since students from the beginning studied the angles relationship as a result of parallel lines cut off by the transverse lines, a dotted line was given to evoke their memories of the shape. In addition, students can also understand that modified forms come from forms they have learned previously. Thus, students are not just doing what is in the example but can interpret what is presented in the example. Furthermore, after students do apperception at the appointed time, the teacher asks questions with the students about what the theorems have been studied in apperception and then affirms that the theorems are needed to solve the geometry problem in the core activities of learning.

3.2. Students Solve Geometry Problems with Worked Example Strategies

In the core activities, students learn to solve geometry problems using the worked example strategies presented in the worksheet. The worked example must be designed in such a way it assists students to understand how the problem is solved [9]. The instruction toward the worked example should understand by students clearly so that they can maximize the knowledge when using a worked example. It is better if the students reading the instructions together, then the teacher gives an explanation until they understood thoroughly. Such an explanation sometimes needed in the worked example. By adding instruction explanations to worked examples give a positive influence on the acquisition of conceptual knowledge [6]. Teachers also explain to the students that they should be able to manipulate the diagram using the auxiliary line to solve a geometry problem. Accordingly, students can associate thoroughly the theorems in the apperception with auxiliary lines to solve the geometry problem. Worked examples provide an expert's problem-solving model, from which students can study and learn [16]. Therefore, students should understand the problem-solving model. How do students understand? They must follow the steps of solution contained in the example sequentially, then understand the explanation at each step. Students may indicate points that are considered important or may also provide additional information with their language.

After understanding the example in detail, then students do the subsequent problem without looking at the example. Paired example and the problem presented on the different page on the worksheet. Such technique in order that students do not depend on the example. But in fact, there are students who see the example repeatedly. Therefore, the teacher plays an important role in reminding the students and motivating them to maximize their ability when understanding the examples and then using them to work on the problem. In addition, during the instruction, it is also important to motivate students to understand meaningfully rather than just memorizing [9]. For a complex problem learning material, students better acquiring the problem and the solutions step individually [21, 22]. When students are solving a problem, firstly they must draw the auxiliary line like in the example. then, students can apply the knowledge of line and angle theorems to find the angle (the goal of the problem). Students should follow the steps as same as on the example.

It is common for students to see examples of confusion, but students can quickly understand the example by learning the information contained in it. Students with sufficient apperception knowledge can understand the example thoroughly and in a relatively short time. However, for students whose does not have enough apperception knowledge usually takes more time to understand the example. So that time allocation also needs to be submitted to the students. Students are given 50 minutes to understand the five examples and do the five problems in the worksheet. However, in practice, all students can complete the work in just 35 minutes. This suggests that the examples provided are easily understood by the students and helpful when working on the questions. That results in line with reviewed studies, that worked example have a beneficial effect on learning (either in terms of performance, time, or both) [23].

3.3. Students Present the Results of Work
Students who finished the worksheets thoroughly may not be able to explain them properly either. When some students presentation in front of the class, there are students who can not explain their answer. The presentation helps students to confirm their answers. Students who have been able to use the worked example properly can help other students who still do not understand. Besides, it is motivating since students would feel satisfied with their learning achievement [9]. In addition, teachers can also provide an affirmation of the key points they must remember in order to solve the other problems without pairing an example.

3.4. Teachers Guide Students to Conclude Learning Outcomes
At the end of the lesson, teachers need to guide students to make conclusions about what they have gained during the learning process. This is to provide an affirmation of the important points that have been learned by students. accordingly, students can keep such knowledge in long-term memory so that they can recall when finding similar geometry problems. The expected conclusion of learning to solve problems about an angle which formed by parallel lines and two or more transverse lines, i.e. students can understand that using manipulation the diagram by creating auxiliary lines can solve the problem of angles formed by parallel lines and transverse lines. In addition, when solving a problem, students can use a variety of ways and choose the easiest way to understand.

4. Conclusion
In grade 7, students have to learn geometry theorems while solving a complex problem, such as the angles formed by parallel lines and transversal lines. Moreover, if the problem is a new material for the students, they may find it difficult because they have insufficient prior knowledge. Therefore, worked examples are suggested for assisting learning. Worked examples can be presented in a worksheet, paired with similar problems. By this way, complex geometry problems can be learned more effectively because the worked example provides a knowledge base of such problem solution.

There are several things that must be considered when creating worked example for geometry, included how to visualize the geometry concept, how to format the picture, and how to design a self-centended example. In the classroom, to teach using this worksheet, teachers should start with an introduction to recall relevant knowledge, then students solve geometry problems with worked example strategies, followed by presentation, clarification and conclusion, guided by the teacher.

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